

Complementarity, quantum erasure and delayed choice with modified Mach-Zehnder interferometers

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Abstract. Often cited dictums in Quantum Mechanics include “observation disturbance causes loss of interference” and “ignorance is interference”. In this paper we propose and describe a series of experiments with modified Mach-Zehnder interferometers showing that one has to be careful when applying such dictums. We are able to show that without interacting in any way with the light quantum (or quanta) expected to behave “wave-like”, interference fringes can be lost by simply gaining (or having the potential to gain) the which-path knowledge. Erasing this information may revive the interference fringes. Delayed choice can be added, arriving to an experiment in line with Wheeler’s original proposal. We also show that ignorance is not always synonym with having the interference fringes. The often-invoked “collapse of the wavefunction” is found to be a non-necessary ingredient to describe our experiments.

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1 Introduction

The famous Einstein-Bohr debates [1] and Einstein’s counter-examples to the then-newly founded Quantum Mechanics treat the loss of interference in a two-slit experiment as a consequence of the perturbation induced by the measuring device. Heisenberg used this fact in his famous *Gedankenexperiment* [2] when he introduced the uncertainty principle: a disturbance based analysis for the observation of an electron in orbit with a (gamma ray) light quantum. Bohr [3] was the first to realize that Quantum Mechanics is more subtle than this “observation caused disturbance induces the uncertainty” dictum and his Complementary Principle refined the so-called wave-particle duality to a new level. This work spurred various theoretical refinements [4, 5] and experimental verifications [6, 7, 8, 9, 10].

A couple of decades after Bohr’s proposal, Wheeler [11, 12] introduced the idea of delayed-choice experiment. One could decide what type of phenomenon is measured *after* the actual measurement was done. In his original *Gedankenexperiment* one could decide to remove or not the second beam splitter in a Mach-Zehnder interferometer after the first light quantum left the first beam splitter, this deciding on the wave-like or particle-like phenomenon that is measured. It took several decades until Jacques *et al.* [13, 14] fully tested Wheeler’s original delayed choice proposal. Previous experiments tested equivalent schemes [15, 16].

Meanwhile the new idea of *Quantum Eraser* was proposed by Scully and Drühl [17]. In fact, it was possible to “erase” a which path information and – surprisingly – revive the interference fringes previously washed out by

the which-path markers [18]. This proposition stirred also an interesting controversy [19, 20, 21], showing how difficult to grasp the predictions of Quantum Mechanics can be. A detailed discussion of the initial proposition can be found in reference [22].

The spectacular evolutions in Quantum Optics (QO) made available sources of single and twin light quanta (via the process of Spontaneous Parametric Down-Conversions - SPDC [23, 24]). Therefore, many fundamental experiments from Quantum Mechanics became feasible in this field (for a review, see for example Steinberg *et al.* [25]). The so-called “high-NOON” states *i.e.* quantum states of the type $|\psi\rangle = 1/\sqrt{2}(|N0\rangle + |0N\rangle)$ became an experimental reality [26, 27, 28, 29] in the last decade. We shall consider this type of maximally entangled quantum states in our experimental setup.

The Mach-Zehnder interferometer (MZI) is often chosen to test various crucial features of the quantum nature of light. Its versatility led to its use in many experiments [30, 31, 32, 33, 34, 35, 36]. With a single light quantum at one input (the other one being kept “dark”), the rate of photo-detection at any of its outputs oscillates as the path-length difference of the interferometer is swept [30] (and obviously no coincident counts are detected). Applying pairs of light quanta at its inputs (*i.e.* allowing fourth-order correlation measurements), shows quantum phenomena impossible to explain with a semi-classical theory. Having two light quanta in the MZI implies also that the spatial frequency of the output interference fringes doubles [31, 36].

Ou *et al.* [37] inserted parametric down-converters in the arms of a MZI and showed that the coincidence counts

between the inner and the outer MZI show a sinusoidal variation as the pump phases are varied. We shall make reference to their work later on. Walborn *et al.* [38] and later Kwiat *et al.* [39] proposed fully-fledged optical quantum erasers, where the interference can be destroyed and revived in function of the measured polarization of the coincident choices. Y-H. Kim *et al.* [40] used a modified Mach-Zehnder interferometer to create an optical version of the delayed-choice quantum eraser. By choosing among the possible coincident detections, one can select the which-path information and lose the interference or, erase this information and regain the interference fringes. A theoretical proposal for a quantum eraser experiment using two MZIs connected via Kerr crystals was done by Hong *et al.* [41].

In this paper, we propose and describe in the formalism of QO several experiments able to show both delayed choice and quantum erasure. Contrary to other experiments, we use a maximally entangled state inside the Mach-Zehnder interferometer followed by the detection of one (of two or more) photons as which-path marker. We show that as soon as we got the which-way information (in German *welcher Weg*), the interference fringes disappear although we *did not interact* in any way with the light quantum (or quanta) supposed to interfere in the MZI. Erasing this information brings back the interference fringes, and since this decision can be done with a space-like separation from the other detection event, we obtain a delayed-choice experiment in line with Wheeler's original proposal.

The “collapse of the wavefunction” picture is also questioned. Bohr's complementarity principle is shown to fully explain all expected experimental results and the picture of “collapse” is found to be merely a mental process of a *particular experimenter* gaining some *particular information* about the measured quantum system.

This paper is organized as follows. In Section 2 we describe the first experiment able to show the particle nature of light. In Section 3, the which-path information is erased and therefore we are able to show the wave nature of light quanta. All delayed choice scenarios and their implications are discussed in detail in Section 4. The idea of “engineered” input states showing no interference fringes in both previously discussed experimental setups is introduced in Section 5. An extension to our experimental setup to three or more braced MZIs is discussed in Section 6. Finally, conclusions are drawn in Section 7.

2 A Mach-Zehnder interferometer with which-path knowledge

We consider the experimental setup depicted in Fig. 1. The Mach-Zehnder interferometer is composed of the beam splitters BS₁ and BS₃, together with the mirrors M₁ and M₂. The beam splitters are assumed identical and are characterized by the transmission (reflection), coefficients T (R). However, in each arm of the interferometer we introduced a beam splitter (denoted BS₄ and, respectively,

BS₅). These two newly introduced beam splitters (assumed identical) are characterized by the coefficients T_1 (transmission) and R_1 (reflection). The delays φ_C and φ_B are voluntarily introduced in the upper and, respectively, lower path of the interferometer. We further assume that with the two delays set to zero the length of the two paths of the MZI are equal. Detectors D_{10} and D_{11} are placed at the two outputs of beam splitter BS₃. Throughout this paper, we shall assume ideal photo-detectors. The notation used to describe this experiment was done for future convenience (see Section 3).

At the inputs labelled “0” and “1” we apply pairs of identical photons, *i.e.* the input state can be written as

$$|\psi_{in}\rangle = |1_0 1_1\rangle = \hat{a}_0^\dagger \hat{a}_1^\dagger |0\rangle \quad (1)$$

where \hat{a}_k^\dagger denotes the creation operator for the mode (port) k . The state $|1_0 1_1\rangle$ denotes a Fock state with one light quantum in both ports 0 and 1 and $|0\rangle$ denotes the vacuum state. The input field operators obey the usual commutation relations $[\hat{a}_l, \hat{a}_k] = [\hat{a}_l^\dagger, \hat{a}_k^\dagger] = 0$ and $[\hat{a}_l, \hat{a}_k^\dagger] = \delta_{lk}$ where δ_{lk} is the Kronecker delta and $l, k = 0, 1$. Imposing the same commutation relations to the output field operators, one ends up with the well-known constraints [42, 43, 44] on the beam splitter

$$|T|^2 + |R|^2 = 1 \quad (2)$$

and

$$RT^* + TR^* = 0 \quad (3)$$

Using the transformation equations for the creation operators

$$\hat{a}_0^\dagger = T\hat{a}_2^\dagger + R\hat{a}_3^\dagger \quad (4)$$

and

$$\hat{a}_1^\dagger = R\hat{a}_2^\dagger + T\hat{a}_3^\dagger \quad (5)$$

we easily find the state vector after BS₁, namely

$$|\psi_{23}\rangle = \sqrt{2}TR(|0_2 2_3\rangle + |2_2 0_3\rangle) + (T^2 + R^2)|1_2 1_3\rangle \quad (6)$$

Throughout this paper, when dealing with a balanced (50/50) beam splitter, we shall use $T = 1/\sqrt{2}$ and $R = i/\sqrt{2}$ [42] implying $R^2 + T^2 = 0$, therefore in Eq. (6) the $|1_2 1_3\rangle$ output state vanishes yielding

$$|\psi_{23}\rangle = \frac{i}{\sqrt{2}}(|0_2 2_3\rangle + |2_2 0_3\rangle) \quad (7)$$

This is called the *antibunching* or HOM effect [42, 45, 46]. For the experiment under discussion, Eq. (7) describes a crucial point: in our interferometer, the light quanta will “bunch” *i.e.* if one light quantum is detected by, say, detector D₆, we know *with certainty* that the other light quantum is in the lower arm of the interferometer. Thus, we have the *welcher Weg* information. However, we got this information without interacting with (or disturbing) *in any way* the second light quantum.

In the following, we will describe the proposed experiment using the standard formalism of QO. Using the fact

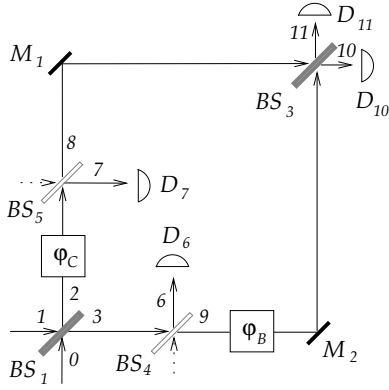


Fig. 1. To the Mach-Zehnder interferometer formed by the beam splitters BS_1 and BS_3 , together with the mirrors M_1 and M_2 a beam splitter was introduced in each arm of the interferometer (denoted BS_4 and, respectively, BS_5). We can have thus the which-path information if any of D_6 or D_7 “clicks”. The delays φ_C and φ_B allow one to modify the path length difference in the MZI.

that an input vacuum state transforms into an output vacuum state, one can compute the output state vector. We first write the input field operators \hat{a}_0^\dagger and \hat{a}_1^\dagger in respect with the output ones. We obtain

$$\begin{aligned} \hat{a}_0^\dagger &= T_1 (T^2 e^{i\varphi_C} + R^2 e^{i\varphi_B}) \hat{a}_{10}^\dagger \\ &+ T_1 T R (e^{i\varphi_C} + e^{i\varphi_B}) \hat{a}_{11}^\dagger + R_1 R \hat{a}_6^\dagger + R_1 T e^{i\varphi_C} \hat{a}_7^\dagger \end{aligned} \quad (8)$$

and

$$\begin{aligned} \hat{a}_1^\dagger &= T_1 T R (e^{i\varphi_C} + e^{i\varphi_B}) \hat{a}_{10}^\dagger \\ &+ T_1 (T^2 e^{i\varphi_B} + R^2 e^{i\varphi_C}) \hat{a}_{11}^\dagger + R_1 T \hat{a}_6^\dagger + R_1 R e^{i\varphi_C} \hat{a}_7^\dagger \end{aligned} \quad (9)$$

Assuming the beam splitters BS_1 and BS_3 to be balanced (50/50), after some calculations, one gets the output¹ state

$$\begin{aligned} |\psi_{out}\rangle &= \frac{T_1^2 e^{i(\varphi_C + \varphi_B)}}{\sqrt{2}} \sin(\Delta\varphi_B) (|2_{10}0_{11}\rangle - |0_{10}2_{11}\rangle) \\ &\quad - T_1^2 e^{i(\varphi_C + \varphi_B)} \cos(\Delta\varphi_B) |1_{10}1_{11}\rangle \\ &+ \frac{iR_1^2}{\sqrt{2}} (|2_{60}0_7\rangle + e^{i\varphi_C} |0_{62}0_7\rangle) + \frac{T_1 R_1}{\sqrt{2}} (-e^{i\varphi_B} |1_61_{10}\rangle \\ &\quad + ie^{i\varphi_C} |1_71_{10}\rangle + ie^{i\varphi_B} |1_61_{11}\rangle - e^{i\varphi_C} |1_71_{11}\rangle) \end{aligned} \quad (10)$$

where we denoted $\Delta\varphi_B = \varphi_B - \varphi_C$. Using Eq. (10) allows us to compute various detection probabilities at the output ports. For example, the probability of coincident counts at detectors D_{10} and D_{11} is given by

$$P_{10-11} = |\langle 1_{10}1_{11} | \psi_{out} \rangle|^2 = |T_1|^4 \cos^2(\Delta\varphi_B) \quad (11)$$

¹ Strictly speaking we should have written in Eq. (10) $|0_60_72_{10}0_{11}\rangle$ instead of $|2_{10}0_{11}\rangle$, $|0_60_70_{10}2_{11}\rangle$ instead of $|0_{10}2_{11}\rangle$ etc. In order to keep the notation simple, we preferred to denote explicitly only the two modes that appeared in the calculation of the respective terms.

showing indeed the interference pattern frequency expected from a MZI with two simultaneously impinging light quanta at its input [31]. Since none of the detectors D_6 and D_7 “clicked”, we have no which-path information, therefore this interference pattern should come as no surprise².

One could try to spy on the *welcher Weg* information by monitoring the coincident counts between one of the inner detectors (D_6 or D_7) and one of the outer ones (D_{10} or D_{11}). If we consider a detection event at detector D_6 , we could naively assume that after BS_4 and BS_5 the quantum state inside the MZI is $1/\sqrt{2}(|1_80_9\rangle + e^{i\varphi_C}|0_81_9\rangle)$, therefore we expect interference fringes on checking the counts at say, D_{10} (conditioned on a detection at D_6) similar to other experiments using a MZI with a single quantum of light at its input [30]. But this is not what we find if we project the output state Eq. (10) onto the state $|1_61_{10}\rangle$. Indeed, we get

$$P_{6-10} = |\langle 1_61_{10} | \psi_{out} \rangle|^2 = \frac{|T_1 R_1|^2}{2} \quad (12)$$

and we find no interference fringes on varying the interferometer’s arm length difference. At a more careful look, this should come at no surprise since by detecting the first light quantum at the detector D_6 we got the *which-path information* for the second light quantum: it took, *with certainty*, the lower arm of the interferometer. Therefore, we could assume that the state inside the MZI after this detection “collapsed” to $|0_81_9\rangle$. Or, we can simply take the piece of information “detection event at D_6 ” and correspondingly perform a state reduction yielding the same result.

As discussed in Appendix A, by simply ignoring the inner detectors D_6 and D_7 and focusing on the outer ones, we get no additional information.

One might wonder if the interference fringes could be somehow restored while still reading a detection at D_6 (or D_7). The answer is affirmative if we modify the experimental setup, so that the which-path information given by these detectors is *erased*. For example, if in Eq. (12) instead of projecting the state vector $|\psi_{out}\rangle$ onto $|1_61_{10}\rangle$ we compute its projection onto $1/\sqrt{2}(|1_61_{10}\rangle - i|1_71_{10}\rangle)$, we would get

$$\begin{aligned} P'_{6-10} &= \left| \frac{1}{\sqrt{2}} (\langle 1_61_{10} | + i \langle 1_71_{10} |) \psi_{out} \rangle \right|^2 \\ &\sim \cos^2 \left(\frac{\Delta\varphi_B}{2} \right) \end{aligned} \quad (13)$$

and the interference fringes are revived. This is equivalent to inserting a new beam splitter between detectors D_6 and D_7 and in the paths coming from the beam splitters BS_4 and BS_5 . This remark takes us to the next experimental setup.

² It is noteworthy that the coincidence probability at detectors D_6 and D_7 is zero. At a closer look, this is simply the HOM or antibunching effect.

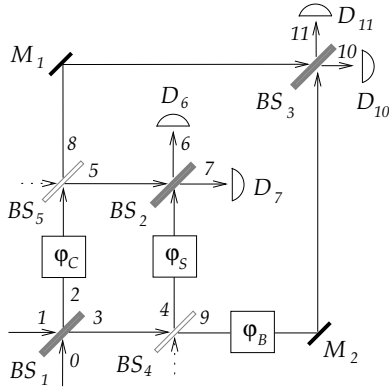


Fig. 2. Two braced Mach-Zehnder interferometers. φ_C accounts for a voluntarily introduced delay common to both MZIs, while the newly introduced φ_S allows one to modify the path length difference in the small MZI only.

3 Two braced Mach-Zehnder interferometers

The beam splitter BS_2 is added to the experimental setup as depicted in Fig. 2. We also add a delay denoted φ_S , able to modify the path length difference only in the “small” MZI, without affecting the initial one. A detection at any of D_6 or D_7 cannot provide anymore the which-path information for the second light quantum, therefore we expect the interference to be restored.

We assume the state vector given by Eq. (1) at the input of BS_1 . In order to find the output state vector, we again express the input field operators \hat{a}_0^\dagger and \hat{a}_1^\dagger in respect with the output ones. We obtain

$$\hat{a}_0^\dagger = T_1 \left((T^2 e^{i\varphi_C} + R^2 e^{i\varphi_B}) \hat{a}_{10}^\dagger + TR (e^{i\varphi_C} + e^{i\varphi_B}) \hat{a}_{11}^\dagger \right) + R_1 \left(TR (e^{i\varphi_C} + e^{i\varphi_S}) \hat{a}_6^\dagger + (T^2 e^{i\varphi_C} + R^2 e^{i\varphi_S}) \hat{a}_7^\dagger \right) \quad (14)$$

and

$$\hat{a}_1^\dagger = T_1 \left(TR (e^{i\varphi_C} + e^{i\varphi_B}) \hat{a}_{10}^\dagger + (T^2 e^{i\varphi_B} + R^2 e^{i\varphi_C}) \hat{a}_{11}^\dagger \right) + R_1 \left((T^2 e^{i\varphi_S} + R^2 e^{i\varphi_C}) \hat{a}_6^\dagger + TR (e^{i\varphi_C} + e^{i\varphi_S}) \hat{a}_7^\dagger \right) \quad (15)$$

If the beam splitters BS_1 , BS_2 and BS_3 are balanced, after a series of long but straightforward calculations we obtain the output state as

$$\begin{aligned} |\psi_{out}\rangle = & T_1^2 e^{i(\varphi_C + \varphi_B)} \left(\frac{\sin(\Delta\varphi_B)}{\sqrt{2}} (|2_{10}0_{11}\rangle - |0_{10}2_{11}\rangle) \right. \\ & \left. - \cos(\Delta\varphi_B) |1_{10}1_{11}\rangle \right) + R_1^2 e^{i(\varphi_C + \varphi_S)} \left(-\cos(\Delta\varphi_S) |1_6 1_7\rangle \right. \\ & \left. + \frac{\sin(\Delta\varphi_S)}{\sqrt{2}} (|0_6 2_7\rangle - |2_6 0_7\rangle) \right) \\ & + T_1 R_1 e^{i\frac{\varphi_B + \varphi_S}{2}} \left(\cos\left(\frac{\Delta\varphi_B + \Delta\varphi_S}{2}\right) |1_6 1_{10}\rangle \right. \\ & + \sin\left(\frac{\Delta\varphi_B + \Delta\varphi_S}{2}\right) |1_6 1_{11}\rangle - \sin\left(\frac{\Delta\varphi_B + \Delta\varphi_S}{2}\right) |1_7 1_{10}\rangle \\ & \left. + \cos\left(\frac{\Delta\varphi_B + \Delta\varphi_S}{2}\right) |1_7 1_{11}\rangle \right) \quad (16) \end{aligned}$$

where $\Delta\varphi_S = \varphi_S - \varphi_C$. For the probability of coincident counts $P_{10-11} = |\langle 1_{10}1_{11} | \psi_{out} \rangle|^2$ we obtain again the result from Eq. (11). The coincidence probability at the inner MZI detectors yields now

$$P_{6-7} = |\langle 1_6 1_7 | \psi_{out} \rangle|^2 = |R_1|^4 \cos^2(\Delta\varphi_S) \quad (17)$$

showing an interference pattern having the same spatial frequency as the outer MZI, consistent with having two light quanta in the interferometer.

This time however, the “cross” probabilities of coincident counts also show interference patterns because no detection at any of D_6 or D_7 can provide the which-way information. Indeed, computing again the probability of coincident counts at detectors D_6 and D_{10} yields

$$P_{6-10} = |T_1 R_1|^2 \cos^2\left(\frac{\Delta\varphi_B + \Delta\varphi_S}{2}\right) \quad (18)$$

showing an interference pattern. However, the spatial frequency of this interference pattern is halved compared to Eqs. (11) and (17), consistent with having one light quantum in each MZI. This result is consistent with Ou *et al.* [37] where they found the spatial frequency of the interference fringes half the pump frequency.

It is noteworthy that in Eq. (18) we have $P_{6-10} \sim 1/2(1 + \cos(\varphi_B + \varphi_S - 2\varphi_C))$ i.e. the delay element φ_C “sees” two light quanta.

4 Delayed-choice quantum eraser

Similar to what was performed in delayed choice experiments [13, 14, 15], the beam splitters BS_2 and BS_3 and the corresponding detectors can be put further apart, in order to ensure space-like separation between detection events. Using electro-optical couplers and polarized light for example [13, 14], we could “insert” and “remove” the beam splitter BS_2 at will.

Focusing again on the experiment described in Section 2, detector D_6 can be sufficiently far apart so that the detection event would be performed after the second light quantum left the MZI (that is BS_3) and heading towards one of the detectors D_{10} or D_{11} . Therefore, this (second) light quantum could not be “informed” about the “collapse of the wavefunction” due to a detection event at D_6 and consequently its expected particle-like behavior inside the MZI.

Moreover, the light quantum to be detected by D_6 might arrive there after passing the beam splitter BS_2 (“inserted” in a delayed-choice manner and being at a distance ensuring that the insertion and detection events happen after the other light quantum left BS_3). This is the experimental setup discussed in Section 3. Therefore, the state inside the outer MZI should not “collapse” to $|0_8 1_9\rangle$ as discussed earlier, but would “continue” to be $1/\sqrt{2}(e^{i\varphi_B}|0_8 1_9\rangle + e^{i\varphi_C}|1_8 0_9\rangle)$ implying interference fringes on varying the delays φ_B and φ_C .

We could rightfully ask ourselves the question if any “collapse” is happening after all. To quote Englert, Scully

and Walther [20] “a state reduction must be performed whenever we wish to account for new information about the system”. In our case, the detection event at D_6 (or D_7) yields this new information, therefore the “collapse of the wavefunction” is merely a mental process, not a physical one.

Finally, nobody “informed” the light quanta how to behave: the experimenter(s) simply *selected* and *measured* a certain point of view from our (quantum) system.

5 Engineered states

It is commonplace to think that if we apply (classical or non-classical) light at the input of a MZI, interference fringes will be present at its output on varying the interferometer’s arm length difference. But this has not to be always so, at least not for a *Gedankenexperiment*. We can create (non-classical) states of light that yield no interference fringes. For example, if we apply the input state

$$|\psi_{in}\rangle = \frac{1}{2} \left(|2_0 0_1\rangle - |0_0 2_0\rangle - i\sqrt{2}|1_0 1_1\rangle \right) \quad (19)$$

to the beam splitter BS_1 we find

$$|\psi_{23}\rangle = |2_2 0_3\rangle \quad (20)$$

at its output, in other words we have *with certainty* two light quanta in the upper path and none in the lower one. This state is fundamentally different from Eq. (7), where we have the two light quanta in a coherent superposition of being both in the lower and in the upper arm of the interferometer. Using this input state and Eqs. (14) and (15) one gets the output state

$$\begin{aligned} |\psi_{out}\rangle = & \frac{T_1^2}{2} \left(|2_{10} 0_{11}\rangle - |0_{10} 2_{11}\rangle + i\sqrt{2}|1_{10} 1_{11}\rangle \right) \\ & + \frac{R_1^2}{2} \left(-|2_6 0_7\rangle + |0_6 2_7\rangle + i\sqrt{2}|1_6 1_7\rangle \right) \\ & + T_1 R_1 \left(i|1_6 1_{10}\rangle + |1_6 1_{11}\rangle + |1_7 1_{10}\rangle + i|1_7 1_{11}\rangle \right) \end{aligned} \quad (21)$$

showing indeed, no interference fringes although we are in a “wave-like” experimental setup and did not disturb in any way the light quanta inside our interferometer. However, since Eq. (20) allows one – at least in principle – to have the *which-path knowledge*, interference cannot be present.

6 Three or more braced interferometers

We consider now the extension of our previous experiments with a new stage of beam splitters and detectors as depicted in Fig. 3. We further assume that the newly introduced beam splitters $BS_{4'}$ and $BS_{5'}$ are identical and are characterized by the transmission (reflection) coefficients T_1 (R_1). The balanced (50/50) beam splitter $BS_{2'}$ can be inserted or removed at will, allowing delayed choice experiments as discussed earlier.

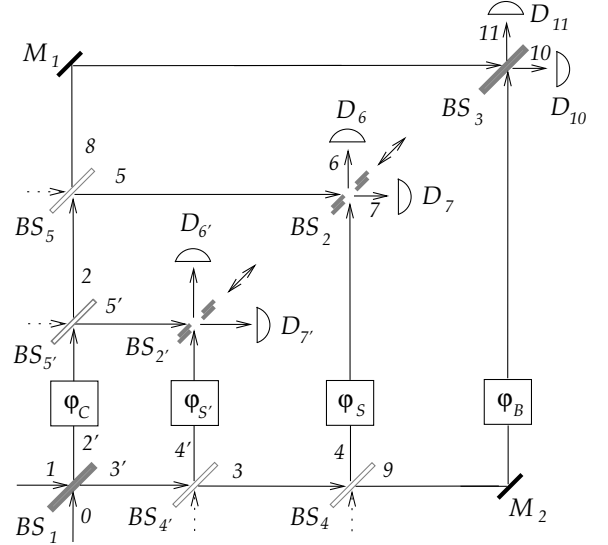


Fig. 3. Three braced Mach-Zehnder interferometers. The newly introduced detectors $D_{6'}$ and $D_{7'}$ allow one to chose between which-path information or wave-like behavior in the MZI composed of BS_1 , $BS_{2'}$, $BS_{4'}$ and $BS_{5'}$. $\varphi_{S'}$ accounts for the path length adjustments in this interferometer.

We need a “high NOON” state (with $N = 3$ in this scenario) after BS_1 , therefore we create for the purpose of our *Gedankenexperiment* the input state,

$$\begin{aligned} |\psi_{in}\rangle = & \frac{1+i}{4} \left(|3_0 0_1\rangle + |0_0 3_1\rangle \right. \\ & \left. - \sqrt{3}|2_0 1_1\rangle - \sqrt{3}|1_0 2_1\rangle \right) \end{aligned} \quad (22)$$

so that after BS_1 the state vector is indeed

$$|\psi_{2'3'}\rangle = \frac{1}{\sqrt{2}} (|3_2 0_3\rangle + |0_2 3_3\rangle) \quad (23)$$

We will focus only on the scenario when one of the detectors $D_{6'}$ or $D_{7'}$ clicks once. If the experimenter measuring at the detectors $D_{6'}$ – $D_{7'}$ decides to erase the which-path information (*i.e.* $BS_{2'}$ is inserted), the state after the beam splitters $BS_{4'}$ and $BS_{5'}$ is again given by Eq. (7) and all scenarios described in Sections 2 and 3 remain available to the experimenter(s) measuring at the detectors D_6 – D_7 and D_{10} – D_{11} . For example, if all experimenters agree to measure wave-like properties, the triple coincidence probability at detectors $D_{6'}$, D_6 and D_{10} is found to be

$$\begin{aligned} P_{6'-6-10} = & \frac{3}{4} |T_1 T_{1'} R_1 R_{1'}|^2 \\ & \cdot \left(1 + \sin(3\varphi_C - \varphi_B - \varphi_S - \varphi_{S'}) \right) \end{aligned} \quad (24)$$

showing indeed, interference fringes while varying any of the delay elements. However, if the experimenter measuring at the detectors $D_{6'}$ – $D_{7'}$ decides to remove $BS_{2'}$ and therefore obtain the which-path information, no interference could be expected by correlating this measurement with the results obtained by the other experimenter(s).

Again, we can invoke the “collapse of the wavefunction” after $BS_{4'}$ and $BS_{5'}$ to Eq. (20) (assuming that $D_{7'}$ clicked) or simply accept that we actually *selected* and *measured* a particular point of view.

If the experimenter measuring at the detectors $D_{6'}$ – $D_{7'}$ is rather secretive and does not wish to reveal his/her experimental setup, a density matrix approach as discussed in Appendix A applies to the double detection rates at any of D_6 , D_7 , D_{10} and D_{11} . Therefore the experimenter(s) measuring at detectors $D_6 - D_7$ and $D_{10} - D_{11}$ might grow frustrated that although the which-path information is not available to them, their measurements show no interference fringes whatsoever. Their error is that they ignore a part of the maximally entangled wavevector given by Eq. (23). Had they used as input state Eq. (1), their “cross” probability of coincident counts (*e.g.* P_{6-10}) would have shown interference fringes³.

Extension to an even larger number of interferometers can be done in a straightforward manner. If we use a carefully chosen state at the input of BS_1 so that at its output the state is

$$|\psi_{2(N-2)3(N-2)}\rangle = \frac{1}{\sqrt{2}} \left(|N_{2(N-2)}0_{3(N-2)}\rangle + |0_{2(N-2)}N_{3(N-2)}\rangle \right) \quad (25)$$

with $N > 3$ and we introduced in our experiment the beam splitters $BS_{k''}$, $BS_{k'''} \dots BS_{k(N-2)}$ with $k = 2, 4, 5$ and detectors $D_{j''}$, $D_{j'''} \dots D_{j(N-2)}$ with $j = 6, 7$.

A N^{th} order coincidence count at any detector pair $D_{10} - D_{11}$, $D_6 - D_7$, $D_{6'} - D_{7'}$, $\dots D_{6(N-2)} - D_{7(N-2)}$ can yield an interference pattern if the corresponding beam splitter (*i.e.* BS_3 , BS_2 , $BS_{2'}$ etc.) is inserted. A cooperation among $M \leq N$ experimenters can also yield interference for any N^{th} order coincident count⁴, with the constraint that all M experimenters make a “wave-like” measurement. Moreover, a single secretive experimenter, not wishing to share his/her information with the other experimenters forces them to measure only statistical mixtures for any single, double, triple, $\dots (N-1)^{th}$ order (coincidence) counts.

All these situations are summarized in Table 1.

7 Conclusions

In this paper we discussed various experimental setups involving modified Mach-Zehnder interferometers showing both the particle and the wave nature of light. We were able to show that if a maximally entangled state is used,

³ Of course, they could also decide to register only triple coincidences (*e.g.* detections at $D_6 - D_7 - D_{10}$, $D_6 - D_{10} - D_{11}$ etc.). The secretive experimenter measuring at the detectors $D_{6'} - D_{7'}$ is therefore “left out” and his/her experimental setup plays no role.

⁴ By “ N^{th} order coincident count” we mean a coincident count among the M cooperative users that adds up to N . For example in Eq. (24) we had $N = 3$ and three experimenters, each one registering a single count.

Table 1. The various scenarios with N -photon states inside N braced Mach-Zehnder interferometers (similar to Fig. 3); BCP stands for “Bohr’s complementary principle” and ISK for “incomplete system knowledge”. Each experimenter controls a pair of detectors.

Path distinguishable?	Cooperative experimenters	Non cooperative experimenters	Related concepts
	Interference pattern?		
yes, for all experimenters	no interference possible for any single/coincident counts for any of the experimenters		BCP
no, for none of the experimenters	for any N^{th} order coincident counts distributed among all experimenters	only N^{th} order coincident counts for a given experimenter	BCP/ISK
not for $M \leq N$ experimenters	for any N^{th} order coincident counts distributed among M experimenters	only N^{th} order coincident counts for a given experimenter (among M)	BCP/ISK

the interference disappears if we *select* events where one light quantum gives the *which-path information*. The interference disappears *without disturbing* in any way the remaining light quantum (quanta). Interference can be revived if we *erase* the which-path information. A delayed-choice version of the experiment can be performed, when the “collapse of the wavefunction” inside one MZI would be determined by the future decision on the erasure of a which-path information. Extensions to three or more braced interferometers has also been discussed.

As a final conclusion, Quantum Mechanics gives us a full and coherent description of the results of our experiments, out of which, the experimenter *selects* a particular *point of view*. We did not need to invoke, at any point, the “collapse of the wavefunction” picture to explain the obtained results.

A Density matrix formalism approach

We assume two experimenters, the first one controlling detectors $D_6 - D_7$ and the second one detectors $D_{10} - D_{11}$.

The following question might arise: if the first experimenter does not wish to communicate his/her experimental setup, could the second experimenter get this information while monitoring the detection rates at D_{10} and/or D_{11} ?

In order to answer this question, we shall use the density matrix approach. Assuming the experimental setup from Fig. 1, the output state vector is given by Eq. (10) therefore we have the output density matrix

$$\hat{\rho}_{out} = |\psi_{out}\rangle\langle\psi_{out}| \quad (26)$$

If we are forced to ignore detectors D_6 and D_7 , the output density matrix $\hat{\rho}_{out}$ has to be traced over the modes 6 and

7 yielding the reduced density matrix

$$\hat{\rho}_{10,11} = \text{Tr}_{6,7} \{ \hat{\rho}_{out} \} = \sum_{m,n=0}^{\infty} \langle m_6 n_7 | \psi_{out} \rangle \langle \psi_{out} | m_6 n_7 \rangle \quad (27)$$

and after some calculations we arrive at the expression

$$\begin{aligned} \hat{\rho}_{10,11} = & \frac{|T_1|^4}{2} \sin^2(\Delta\varphi_B) \left(|2_{10}0_{11}\rangle \langle 2_{10}0_{11}| \right. \\ & + |0_{10}2_{11}\rangle \langle 0_{10}2_{11}| \Big) + |T_1|^4 \cos^2(\Delta\varphi_B) |1_{10}1_{11}\rangle \langle 1_{10}1_{11}| \\ & + |R_1|^4 |0_{10}0_{11}\rangle \langle 0_{10}0_{11}| + |T_1 R_1|^2 \left(|1_{10}0_{11}\rangle \langle 1_{10}0_{11}| \right. \\ & \left. + |0_{10}1_{11}\rangle \langle 0_{10}1_{11}| \right) \quad (28) \end{aligned}$$

We are now able to compute any photo-count probability at D_{10} and/or D_{11} . For example, computing the probability of coincident counts at the detectors D_{10} and D_{11} we get

$$P_{10-11} = \text{Tr} \left\{ \hat{a}_{10}^\dagger \hat{a}_{10} \hat{a}_{11}^\dagger \hat{a}_{11} \hat{\rho}_{10,11} \right\} = |T_1|^4 \cos^2(\Delta\varphi_B) \quad (29)$$

and we revisit the result from Eq. (11). However, one could speculate that the single detection rates at any of the detectors D_{10} or D_{11} could yield some supplementary information. Therefore, we compute the single detection rate at, for example, detector D_{10} , yielding

$$P_{10} = \text{Tr} \left\{ \hat{a}_{10}^\dagger \hat{a}_{10} \hat{\rho}_{10,11} \right\} = |T_1|^4 + |T_1 R_1|^2 \quad (30)$$

and no interference fringe variation can be found by varying any arm length difference. But we could blame this result on the “particle-like” experimental setup from Fig. 1.

Therefore, we now assume that the experimenter measuring at detectors $D_6 - D_7$ changed his/her setup according to Fig. 2. We have to compute the density matrix starting from Eq. (16) and again, trace over the inner detectors D_6 and D_7 . After computing the partial trace from Eq. (27) we get the same result Eq. (28) for $\hat{\rho}_{10,11}$. Therefore, in the “wave-like” experimental setup we find the same single detection rate P_{10} given by Eq. (30). We can discard now the speculation done previously: no information about the experimental setup at the inner detectors D_6 and D_7 can be retrieved from any measurement at the outer MZI.

One could wonder if this result is consistent. The experimenter at the inner detectors has no which-path information, therefore the dictum “ignorance is interference” should apply. However, from Eq. (30) it is clear that it does not.

Actually, by ignoring the fact that we started from an *entangled state* and considering only partial measurements, we are bound to find only statistical mixtures. The state vector $|\psi_{out}\rangle$ describes the whole system (*i.e.* both experimenters) and only the knowledge of their *global state* can yield the interference fringes from Eq. (18).

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